NATURAL OPERATIONS IN DIFFERENTIAL GEOMETRY

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Springer-Verlag, 1993
Orders to: Springer-Verlag Heidelberg, Tiergartenstr. 17, D-69121 Heidelberg, Germany, tel. x-49-6221-487-0.
vi + 434 pages. Hardcover ISBN 3-540-56235-4, ISBN 0-387-56235-4.
DM 138,-; L64,50; FF 520,-; Lit. 152.420; öS 1076,40; sFr 121,50;
U.S. Customers: For Price Information please contact SV New York (201)348-4033 (Toll Free: 1 (800)SPRINGER)
Email orders to: orders@springer.de

This is a shortened version containing only the table of contents, the preface, and the introductions to all chapters, and the index.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

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PREFACE

The aim of this work is threefold:

First it should be a monographical work on natural bundles and natural operators in differential geometry. This is a field which every differential geometer has met several times, but which is not treated in detail in one place. Let us explain a little, what we mean by naturality.

Exterior derivative commutes with the pullback of differential forms. In the background of this statement are the following general concepts. The vector bundle $\Lambda^k T^*M$ is in fact the value of a functor, which associates a bundle over M to each manifold M and a vector bundle homomorphism over f to each local diffeomorphism f between manifolds of the same dimension. This is a simple example of the concept of a natural bundle. The fact that the exterior derivative d transforms sections of $\Lambda^k T^*M$ into sections of $\Lambda^{k+1} T^*M$ for every manifold M can be expressed by saying that d is an operator from $\Lambda^k T^*M$ into $\Lambda^{k+1} T^*M$. That the exterior derivative d commutes with local diffeomorphisms now means, that d is a natural operator from the functor $\Lambda^k T^*$ into functor $\Lambda^{k+1} T^*$. If $k > 0$, one can show that d is the unique natural operator between these two natural bundles up to a constant. So even linearity is a consequence of naturality. This result is archetypical for the field we are discussing here. A systematic treatment of naturality in differential geometry requires to describe all natural bundles, and this is also one of the undertakings of this book.

Second this book tries to be a rather comprehensive textbook on all basic structures from the theory of jets which appear in different branches of differential geometry. Even though Ehresmann in his original papers from 1951 underlined the conceptual meaning of the notion of an r-jet for differential geometry, jets have been mostly used as a purely technical tool in certain problems in the theory of systems of partial differential equations, in singularity theory, in variational calculus and in higher order mechanics. But the theory of natural bundles and natural operators clarifies once again that jets are one of the fundamental concepts in differential geometry, so that a thorough treatment of their basic properties plays an important role in this book. We also demonstrate that the central concepts from the theory of connections can very conveniently be formulated in terms of jets, and that this formulation gives a very clear and geometric picture of their properties.

This book also intends to serve as a self-contained introduction to the theory of Weil bundles. These were introduced under the name 'les espaces des points proches' by A. Weil in 1953 and the interest in them has been renewed by the recent description of all product preserving functors on manifolds in terms of products of Weil bundles. And it seems that this technique can lead to further interesting results as well.

Third in the beginning of this book we try to give an introduction to the fundamentals of differential geometry (manifolds, flows, Lie groups, differential forms, bundles and connections) which stresses naturality and functoriality from the beginning and is as coordinate free as possible. Here we present the Frölicher-Nijenhuis bracket (a natural extension of the Lie bracket from vector fields to

vector valued differential forms) as one of the basic structures of differential geometry, and we base nearly all treatment of curvature and Bianchi identities on it. This allows us to present the concept of a connection first on general fiber bundles (without structure group), with curvature, parallel transport and Bianchi identity, and only then add G-equivariance as a further property for principal fiber bundles. We think, that in this way the underlying geometric ideas are more easily understood by the novice than in the traditional approach, where too much structure at the same time is rather confusing. This approach was tested in lecture courses in Brno and Vienna with success.

A specific feature of the book is that the authors are interested in general points of view towards different structures in differential geometry. The modern development of global differential geometry clarified that differential geometric objects form fiber bundles over manifolds as a rule. Nijenhuis revisited the classical theory of geometric objects from this point of view. Each type of geometric objects can be interpreted as a rule F transforming every m-dimensional manifold M into a fibered manifold $FM \to M$ over M and every local diffeomorphism $f : M \to N$ into a fibered manifold morphism $Ff : FM \to FN$ over f. The geometric character of F is then expressed by the functoriality condition $F(g \circ f) = Fg \circ Ff$. Hence the classical bundles of geometric objects are now studied in the form of the so called lifting functors or (which is the same) natural bundles on the category $\mathcal{M}f_m$ of all m-dimensional manifolds and their local diffeomorphisms. An important result by Palais and Terng, completed by Epstein and Thurston, reads that every lifting functor has finite order. This gives a full description of all natural bundles as the fiber bundles associated with the r-th order frame bundles, which is useful in many problems. However in several cases it is not sufficient to study the bundle functors defined on the category $\mathcal{M}f_m$. For example, if we have a Lie group G, its multiplication is a smooth map $\mu : G \times G \to G$. To construct an induced map $F\mu : F(G \times G) \to FG$, we need a functor F defined on the whole category $\mathcal{M}f$ of all manifolds and all smooth maps. In particular, if F preserves products, then it is easy to see that $F\mu$ endows FG with the structure of a Lie group. A fundamental result in the theory of the bundle functors on $\mathcal{M}f$ is the complete description of all product preserving functors in terms of the Weil bundles. This was deduced by Kainz and Michor, and independently by Eck and Luciano, and it is presented in chapter VIII of this book. At several other places we then compare and contrast the properties of the product preserving bundle functors and the non-product-preserving ones, which leads us to interesting geometric results. Further, some functors of modern differential geometry are defined on the category of fibered manifolds and their local isomorphisms, the bundle of general connections being the simplest example. Last but not least we remark that Eck has recently introduced the general concepts of gauge natural bundles and gauge natural operators. Taking into account the present role of gauge theories in theoretical physics and mathematics, we devote the last chapter of the book to this subject.

If we interpret geometric objects as bundle functors defined on a suitable category over manifolds, then some geometric constructions have the role of natural transformations. Several others represent natural operators, i.e. they map sec-

tions of certain fiber bundles to sections of other ones and commute with the action of local isomorphisms. So geometric means natural in such situations. That is why we develop a rather general theory of bundle functors and natural operators in this book. The principal advantage of interpreting geometric as natural is that we obtain a well-defined concept. Then we can pose, and sometimes even solve, the problem of determining all natural operators of a prescribed type. This gives us the complete list of all possible geometric constructions of the type in question. In some cases we even discover new geometric operators in this way.

Our practical experience taught us that the most effective way how to treat natural operators is to reduce the question to a finite order problem, in which the corresponding jet spaces are finite dimensional. Since the finite order natural operators are in a simple bijection with the equivariant maps between the corresponding standard fibers, we can apply then several powerful tools from classical algebra and analysis, which can lead rather quickly to a complete solution of the problem. Such a passing to a finite order situation has been of great profit in other branches of mathematics as well. Historically, the starting point for the reduction to the jet spaces is the famous Peetre theorem saying that every linear support non-increasing operator has locally finite order. We develop an essential generalization of this technique and we present a unified approach to the finite order results for both natural bundles and natural operators in chapter V.

The primary purpose of chapter VI is to explain some general procedures, which can help us in finding all the equivariant maps, i.e. all natural operators of a given type. Nevertheless, the greater part of the geometric results is original. Chapter VII is devoted to some further examples and applications, including Gilkey's theorem that all differential forms depending naturally on Riemannian metrics and satisfying certain homogeneity conditions are in fact Pontryagin forms. This is essential in the recent heat kernel proofs of the Atiyah Singer Index theorem. We also characterize the Chern forms as the only natural forms on linear symmetric connections. In a special section we comment on the results of Kirillov and his colleagues who investigated multilinear natural operators with the help of representation theory of infinite dimensional Lie algebras of vector fields. In chapter X we study systematically the natural operators on vector fields and connections. Chapter XI is devoted to a general theory of Lie derivatives, in which the geometric approach clarifies, among other things, the relations to natural operators.

The material for chapters VI, X and sections 12, 30–32, 47, 49, 50, 52–54 was prepared by the first author (I.K.), for chapters I, II, III, VIII by the second author (P.M.) and for chapters V, IX and sections 13–17, 33, 34, 48, 51 by the third author (J.S.). The authors acknowledge A. Cap, M. Doupovec, and J. Janyška, for reading the manuscript and for several critical remarks and comments and A. A. Kirillov for commenting section 34.

The joint work of the authors on the book has originated in the seminar of the first two authors and has been based on the common cultural heritage of Middle Europe. The authors will be pleased if the reader realizes a reflection of those traditions in the book.

CHAPTER I.
MANIFOLDS AND LIE GROUPS

In this chapter we present an introduction to the basic structures of differential geometry which stresses global structures and categorical thinking. The material presented is standard - but some parts are not so easily found in text books: we treat initial submanifolds and the Frobenius theorem for distributions of non constant rank, and we give a very quick proof of the Campbell - Baker - Hausdorff formula for Lie groups. We also prove that closed subgroups of Lie groups are Lie subgroups.

CHAPTER II.
DIFFERENTIAL FORMS

This chapter is still devoted to the fundamentals of differential geometry, but here the deviation from the standard presentations is already large. In the section on vector bundles we treat the Lie derivative for natural vector bundles, i.e. functors which associate vector bundles to manifolds and vector bundle homomorphisms to local diffeomorphisms. We give a formula for the Lie derivative of the form of a commutator, but it involves the tangent bundle of the vector bundle in question. So we also give a careful treatment to this situation. The Lie derivative will be discussed in detail in chapter XI; here it is presented in a somewhat special situation as an illustration of the categorical methods we are going to apply later on. It follows a standard presentation of differential forms and a thorough treatment of the Frölicher-Nijenhuis bracket via the study of all graded derivations of the algebra of differential forms. This bracket is a natural extension of the Lie bracket from vector fields to tangent bundle valued differential forms. We believe that this bracket is one of the basic structures of differential geometry (see also section 30), and in chapter III we will base nearly all treatment of curvature and the Bianchi identity on it.

CHAPTER III.
BUNDLES AND CONNECTIONS

We begin our treatment of connections in the general setting of fiber bundles (without structure group). A connection on a fiber bundle is just a projection onto the vertical bundle. Curvature and the Bianchi identity is expressed with the help of the Frölicher-Nijenhuis bracket. The parallel transport for such a general connection is not defined along the whole of the curve in the base in general - if this is the case for all curves, the connection is called complete. We show that every fiber bundle admits complete connections. For complete connections we treat holonomy groups and the holonomy Lie algebra, a subalgebra of the Lie algebra of all vector fields on the standard fiber.

Then we present principal bundles and associated bundles in detail together with the most important examples. Finally we investigate principal connections by requiring equivariance under the structure group. It is remarkable how fast the usual structure equations can be derived from the basic properties of the Frölicher-Nijenhuis bracket. Induced connections are investigated thoroughly - we describe tools to recognize induced connections among general ones.

If the holonomy Lie algebra of a connection on a fiber bundle is finite dimensional and consists of complete vector fields on the fiber, we are able to show, that in fact the fiber bundle is associated to a principal bundle and the connection is induced from an irreducible principal connection (theorem 9.11). This is a powerful generalization of the theorem of Ambrose and Singer.

Connections will be treated once again from the point of view of jets, when we have them at our disposal in chapter IV.

We think that the treatment of connections presented here offers some didactical advantages besides presenting new results: the geometric content of a connection is treated first, and the additional requirement of equivariance under a structure group is seen to be additional and can be dealt with later - so the reader is not required to grasp all the structures at the same time. Besides that it gives new results and new insights. There are naturally appearing connections in differential geometry which are not principal or induced connections: The universal connection on the bundle J^1P/G of all connections of a principal bundle, and also the Cartan connections.

Remarks

The concept of connections on general fiber bundles was formulated at about 1970, see e. g. [Libermann, 73]. The theorem 9.9 that each fiber bundle admits a complete connection is contained in [Wolf, 67], with an incorrect proof. It is an exercise in [Greub-Halperin-Vanstone I, 72, p 314]. The proof given here and

the generalization 9.11 of the Ambrose Singer theorem are from [Michor, 88], see also [Michor, 91], which are also the source for 11.8 and 11.9. The results 11.15 and 11.16 appear here for the first time.

CHAPTER IV.
JETS AND NATURAL BUNDLES

In this chapter we start our systematic treatment of geometric objects and operators. It has become commonplace to think of geometric objects on a manifold M as forming fiber bundles over the base M and to work with sections of these bundles. The concrete objects were frequently described in coordinates by their behavior under the coordinate changes. Stressing the change of coordinates, we can say that local diffeomorphisms on the base manifold operate on the bundles of geometric objects. Since a further usual assumption is that the resulting transformations depend only on germs of the underlying morphisms, we actually deal with functors defined on all open submanifolds of M and local diffeomorphisms between them (let us recall that local diffeomorphisms are globally defined locally invertible maps), see the preface. This is the point of view introduced by [Nijenhuis, 72] and worked out later by [Terng, 78], [Palais, Terng, 77], [Epstein, Thurston, 79] and others. These functors are fully determined by their restriction to any open submanifold and therefore they extend to the whole category $\mathcal{M}f_m$ of m-dimensional manifolds and local diffeomorphisms. An important advantage of such a definition of bundles of geometric objects is that we get a precise definition of geometric operators in the concept of natural operators. These are rules transforming sections of one natural bundle into sections of another one and commuting with the induced actions of local diffeomorphisms between the base manifolds.

In the theory of natural bundles and operators, a prominent role is played by jets. Roughly speaking, jets are certain equivalence classes of smooth maps between manifolds, which are represented by Taylor polynomials. We start this chapter with a thorough treatment of jets and jet bundles, and we investigate the so called jet groups. Then we give the definition of natural bundles and deduce that the r-th order natural bundles coincide with the associated fibre bundles to r-th order frame bundles. So they are in bijection with the actions of the r-th order jet group G^r_m on manifolds. Moreover, natural transformations between the r-th order natural bundles bijectively correspond to G^r_m-equivariant maps. Let us note that in chapter V we deduce a rather general theory of functors on categories over manifolds and we prove that both the finiteness of the order and the regularity of natural bundles are consequences of the other axioms, so that actually we describe all natural bundles here. Next we treat the basic properties of natural operators. In particular, we show that k-th order natural operators are described by natural transformations of the k-th order jet prolongations of the bundles in question. This reduces even the problem of finding finite order natural operators to determining G^r_m-equivariant maps, which will be discussed in chapter VI.

Further we present the procedure of principal prolongation of principal fiber bundles based on an idea of [Ehresmann, 55] and we show that the jet prolongations of associated bundles are associated bundles to the principal prolongations of the corresponding principal bundles. This fact is of basic importance for the theory of gauge natural bundles and operators, the foundations of which will be presented in chapter XII. The canonical form on first order principal prolongation of a principal bundle generalizes the well known canonical form on an r-th order frame bundle. These canonical forms are used in a formula for the first jet prolongation of sections of arbitrary associated fiber bundles, which represents a common basis for several algorithms in different branches of differential geometry. At the end of the chapter, we reformulate a part of the theory of connections from the point of view of jets, natural bundles and natural operators. This is necessary for our investigation of natural operations with connections, but we believe that this also demonstrates the power of the jet approach to give a clear picture of geometric concepts.

Remarks

The development of the theory of natural bundles and operators is described in the preface and in the introduction to this chapter. But let us come back to the jet groups. As mentioned in [Reinhart, 83], it is remarkable how very little of existing Lie group theory applies to them. The results deduced in our exposition are mainly due to [Terng, 78] where the reader can find some more information on the classification of G^r_m-modules. For the first order jet groups, it is very useful to study in detail the properties of irreducible representations, cf. section 34. But in view of 13.15 it is not interesting to extend this approach to the higher orders. The bundle functors on the whole category $\mathcal{M}f$ were first studied by [Janyška, 83]. We shall continue the study of such functors in chapter IX.

The basic ideas from section 15 were introduced in a slightly modified situation by [Ehresmann, 55]. Every principal fiber bundle $p\colon P \to M$ with structure group G determines the associated groupoid PP^{-1} which can be defined as the factor space $P \times P/\sim$ with respect to the equivalence relation $(u, v) \sim (ug, vg)$, $u,\ v \in P,\ g \in G$. Writing uv^{-1} for such an equivalence class, we have two projections $a,\ b\colon PP^{-1} \to M$, $a(uv^{-1}) = p(v)$, $b(uv^{-1}) = p(u)$. If E is a fiber bundle associated with P with standard fiber S, then every $\theta = uv^{-1} \in PP^{-1}$ determines a diffeomorphism $q_u \circ (q_v)^{-1} : E_{a\theta} \to E_{b\theta}$, where $q_v : S \to E_{a\theta}$ and $q_u : S \to E_{b\theta}$ are the 'frame maps' introduced in 10.7. This defines an action of groupoid PP^{-1} on fiber bundle E. The space PP^{-1} is a prototype of a smooth groupoid over M. In [Ehresmann, 55] the r-th prolongation Φ^r of an arbitrary smooth groupoid Φ over M is defined and every action of Φ on a fiber bundle $E \to M$ is extended into an action of Φ^r on the r-th jet prolongation J^rE of $E \to M$. This construction was modified to the principal fiber bundles by [Libermann, 71], [Virsik, 69] and [Kolář, 71b].

The canonical $\mathbb{R}^m$-valued form on the first order frame bundle P^1M is one

of the basic concepts of modern differential geometry. Its generalization to r-th order frame bundles was introduced by [Kobayashi, 61]. The canonical form on W^1P (as well as on W^rP) was defined in [Kolář, 71b] in connection with some local considerations by [Laptev, 69] and [Gheorghiev, 68]. Those canonical forms play an important role in a generalization of the Cartan method of moving frames, see [Kolář, 71c, 73a, 73b, 77].

CHAPTER V.
FINITE ORDER THEOREMS

The purpose of this chapter is to develop a general framework for the theory of geometric objects and operators and to reduce local geometric considerations to finite order problems. In general, the latter is a hard analytical problem and its solution essentially depends on the category in question. Roughly speaking, our methods are efficient when we deal with a sufficiently large class of smooth maps, but they fail e.g. for analytic maps.

We first extend the concepts and results from section 14 to a wider class of categories. Then we present our important analytical tool, a nonlinear generalization of well known Peetre theorem. In section 20 we prove the regularity of bundle functors for a class of categories which includes $\mathcal{M}f$, $\mathcal{M}f_m$, $\mathcal{FM}$, $\mathcal{FM}_m$, $\mathcal{FM}_{m,n}$, and we get near to the finiteness of the order of bundle functors. It remains to deduce estimates on the possible orders of jet groups acting on manifolds. We derive such estimates for the actions of jet groups in the category $\mathcal{FM}_{m,n}$ so that we describe all bundle functors on $\mathcal{FM}_{m,n}$. For $n = 0$ this reproves in a different way the classical results due to [Palais, Terng, 77] and [Epstein, Thurston, 79] on the regularity and the finiteness of the order of natural bundles.

The end of the chapter is devoted to a discussion on the order of natural operators. Also here we essentially profit from the nonlinear Peetre theorem. First of all, its trivial consequence is that every (even not natural) local operator depends on infinite jets only. So instead of natural transformations between the infinite dimensional spaces of sections of the bundles in question, we have to deal with natural transformations between the (infinite) jet prolongations. The full version of Peetre theorem implies that in fact the order is finite on large subsets of the infinite jet spaces and, by naturality, the order is invariant under the action of local isomorphisms on the infinite jets. In many concrete situations the whole infinite jet prolongation happens to be the orbit of such a subset. Then all natural operators from the bundle in question are of finite order and the problem of finding a full list of them can be attacked by the methods developed in the next chapter.

Remarks

The general setting for bundle functors and natural operators extends the original categorical approach to geometric objects and operators due to [Nijenhuis, 72] and we follow mainly [Kolář, 90] and partially [Slovák, 91].

The multilinear version of Peetre theorem, proved in [Cahen, De Wilde, Gutt, 80], seems to be the first non-linear generalization of the famous Peetre theorem, [Peetre, 60]. The study of general nonlinear operators started in [Chrastina, 87] and [Slovák, 87b]. The original aim of the nonlinear version 19.7, first proved in [Slovák, 87b], was the reduction of the problem of finding natural operators to a finite order. The pure analytical results were further generalized and completed in a setting of Hölder-continuous maps and metric spaces in [Slovák, 88] and it became clear that they should help to unify the approach to the finiteness of the orders of both natural operators and bundle functors and to avoid the original manipulation with infinite dimensional Lie algebras, see [Palais, Terng, 77]. Let us remark that nearly all categories over manifolds used in differential geometry are admissible and locally flat, however the verification of the Whitney extendibility might present a serious analytical problem in concrete examples. In the most technical part of the description of bundle functors, i.e. in the proof of the regularity, we mainly follow [Mikulski, 85] which generalizes the original proof due to [Epstein, Thurston, 79] to natural bundles with infinite dimensional values. Let us point out that our proof also applies to continuous regularity of bundle functors on the categories in question with values in infinite dimensional manifolds.

Our sharp estimate on the orders of jet groups acting on manifolds is a generalization of [Zajtz, 87], where similar results are obtained for the full group G^r_m. The results on the order of bundle functors on $\mathcal{FM}_m$ follow some ideas from [Kolář, Slovák, 89] and [Mikulski, 89 a, b]. The methods used in our discussion on the order of natural operators never exploit the regularity of the natural operators which we have incorporated into our definition. So the results of section 23 can be applied to non-regular natural operators which can also be classified in some concrete situations.

CHAPTER VI.
METHODS FOR FINDING NATURAL OPERATORS

We present certain general procedures useful for finding some equivariant maps and we clarify their application by solving concrete geometric problems. The equivariance with respect to the homotheties in $GL(m)$ gives frequently a homogeneity condition. The homogeneous function theorem reads that under certain assumptions a globally defined smooth homogeneous function must be polynomial. In such a case the use of the invariant tensor theorem and the polarization technique can specify the form of the polynomial equivariant map up to such an extend, that all equivariant maps can then be determined by direct evaluation of the equivariance condition with respect to the kernel of the jet projection $G^r_m \to G^1_m$. We first deduce in such a way that all natural operators transforming linear connections into linear connections form a simple 3-parameter family. Then we strengthen a classical result by Palais, who deduced that all linear natural operators $\Lambda^p T^* \to \Lambda^{p+1} T^*$ are the constant multiples of the exterior derivative. We prove that for $p > 0$ even linearity follows from naturality. We underline, as a typical feature of our procedures, that in both cases we first have guaranteed by the results from chapter V that the natural operators in question have finite order. Then the homogeneous function theorem implies that the natural operators have zero order in the first case and first order in the second case. In section 26 we develop the smooth version of the tensor evaluation theorem. As the first application we determine all natural transformations $TT^* \to T^*T$. The result implies that, unlike to the case of cotangent bundle, there is no natural symplectic structure on the tangent bundle.

As an example of a natural operator related with fibered manifolds we discuss the curvature of a general connection. An important tool here is the generalized invariant tensor theorem, which describes all $GL(m) \times GL(n)$-invariant tensors. We deduce that all natural operators of the curvature type are the constant multiples of the curvature and that all such operators on a pair of connections are linear combinations of the curvatures of the individual connections and of the so-called mixed curvature of both connections. The next section is devoted to the orbit reduction. We develop a complete version of the classical reduction theorem for linear symmetric connections and Riemannian metrics, in which the factorization procedure is described in terms of the curvature spaces and the Ricci spaces. The so-called method of differential equations is based on the simple fact that on the Lie algebra level the equivariance condition represents a system of partial differential equations. As an example we deduce that the only first order natural operator transforming Riemannian metrics into linear connections is the Levi-Cività operator. But we apply the method of differential

equations only in the first part of the proof, while in the final step a direct geometric consideration is used.

Remarks

The first version of our systematical approach to the problem of finding natural operators was published in [Kolář, 87b]. In the same paper both geometric results from section 25 are deduced. The smooth version of the tensor evaluation theorem is first presented in this book. Proposition 26.12 was proved by [Kolář, Radziszewski, 88]. The generalized invariant tensor theorem was first used in [Kolář, 87b].

The reduction theorems for symmetric linear connections and pseudoriemannian metrics are classical, see e.g. [Schouten, 54]. Some extensions or reformulations of them are presented in [Lubczonok, 72] and [Krupka, 82]. The method of differential equations is used systematically e.g. in the book [Krupka, Janyška, 90]. The complete version of proposition 29.7 was deduced in [Slovák, 89].

CHAPTER VII.
FURTHER APPLICATIONS

In this chapter we discuss some further geometric problems about different types of natural operators. First we deduce that all natural bilinear operators transforming a vector field and a differential k-form into a differential k-form form a 2-parameter family. This further clarifies the well known relation between Lie derivatives and exterior derivatives of k-forms. From the technical point of view this problem can be considered as a preparatory exercise to the problem of finding all bilinear natural operators of the type of the Frölicher-Nijenhuis bracket. We deduce that in general case all such operators form a 10-parameter family. Then we prove that there is exactly one natural operator transforming general connections on a fibered manifold $Y \to M$ into general connections on its vertical tangent bundle $VY \to M$. Furthermore, starting from some geometric problems in analytical mechanics, we deduce that all first-order natural operators transforming second-order differential equations on a manifold M into general connections on its tangent bundle $TM \to M$ form a one parameter family. Further we study the natural transformations of the jet functors. The construction of the bundle of all r-jets between any two manifolds can be interpreted as a functor J^r on the product category $\mathcal{M}f_m \times \mathcal{M}f$. We deduce that for $r \geq 2$ the only natural transformations of J^r into itself are the identity and the contraction, while for $r = 1$ we have a one-parameter family of homotheties. This implies easily that the only natural transformation of the functor of the r-th jet prolongation of fibered manifolds into itself is the identity. For the second iterated jet prolongation $J^1(J^1Y)$ of a fibered manifold Y we look for an analogy of the canonical involution on the second iterated tangent bundle TTM. We prove that such an exchange map depends on a linear connection on the base manifold and we give a simple list of all natural transformations of this type.

The next section is devoted to some problems from Riemannian geometry. Here we complete our study of natural connections on Riemannian manifolds, we prove the Gilkey theorem on natural differential forms and we find all natural lifts of Riemannian metrics to the tangent bundles. We also deduce that all natural operators transforming linear symmetric connections into exterior forms are generated by the Chern forms. Since there are no natural forms of odd degree, all of them are closed.

In the last section, we present a survey of some results concerning the multilinear natural operators which are based heavily on the (linear) representation theory of Lie algebras. First we treat the naturality over the whole category $\mathcal{M}f_m$, where the main tools come from the representation theory of infinite dimensional algebras of vector fields. At the very end we comment briefly on the

category of conformal (Riemannian) manifolds, which leads to finite dimensional representation theory of some parabolic subalgebras of the Lie algebras of the pseudo orthogonal groups.

Remarks

Proposition 30.4 was proved by [Kolář, Michor, 87]. Proposition 31.1 was deduced in [Kolář, 87a]. The natural transformations $J^r \to J^r$ were determined in [Kolář, Vosmanská, 89]. The exchange map e_Λ from 32.4 was introduced by [Modugno, 89a].

The original proof of the Gilkey theorem on the uniqueness of the Pontryagin forms, [Gilkey, 73], was much more combinatorial and had not used H. Weyl's theorem. Our approach is similar to [Atiyah, Bott, Patodi, 73], but we do not need their polynomiality assumption. The Gilkey theorem was generalized in several directions. For the case of Hermitian bundles and connections see [Atiyah, Bott, Patodi, 73], for oriented Riemannian manifolds see [Stredder, 75], the metrics with a general signature are treated in [Gilkey, 75]. The uniqueness of the Levi-Cività connection among the polynomial conformal natural connections on Riemannian manifolds was deduced by [Epstein, 75]. The classification of the first order liftings of Riemannian metrics to the tangent bundles covers the results due to [Kowalski, Sekizawa, 88], who used the so called method of differential equations in their much longer proof. Our methods originate in [Slovák, 89] and an unpublished paper by W. M. Mikulski.

CHAPTER VIII.
PRODUCT PRESERVING FUNCTORS

We first present the theory of those bundle functors which are determined by local algebras in the sense of A. Weil, [Weil, 51]. Then we explain that the Weil functors are closely related to arbitrary product preserving functors $\mathcal{M}f \to \mathcal{M}f$. In particular, every product preserving bundle functor on $\mathcal{M}f$ is a Weil functor and the natural transformations between two such functors are in bijection with the homomorphisms of the local algebras in question.

In order to motivate the development in this chapter we will tell first a mathematical short story. For a smooth manifold M, one can prove that the space of algebra homomorphisms $\mathrm{Hom}(C^\infty(M,\mathbb{R}),\mathbb{R})$ equals M as follows. The kernel of a homomorphism $\varphi : C^\infty(M,\mathbb{R}) \to \mathbb{R}$ is an ideal of codimension 1 in $C^\infty(M,\mathbb{R})$. The zero sets $Z_f := f^{-1}(0)$ for $f \in \ker\varphi$ form a filter of closed sets, since $Z_f \cap Z_g = Z_{f^2+g^2}$, which contains a compact set Z_f for a function f which is unbounded on each non compact closed subset. Thus $\bigcap_{f\in\ker\varphi} Z_f$ is not empty, it contains at least one point x_0. But then for any $f \in C^\infty(M,\mathbb{R})$ the function $f-\varphi(f)1$ belongs to the kernel of φ, so vanishes on x_0 and we have $f(x_0) = \varphi(f)$.

An easy consequence is that $\mathrm{Hom}(C^\infty(M,\mathbb{R}),C^\infty(N,\mathbb{R})) = C^\infty(N,M)$. So the category of algebras $C^\infty(M,\mathbb{R})$ and their algebra homomorphisms is dual to the category $\mathcal{M}f$ of manifolds and smooth mappings.

But now let $\mathbb{D}$ be the algebra generated by 1 and ε with $\varepsilon^2 = 0$ (sometimes called the algebra of dual numbers or Study numbers, it is also the truncated polynomial algebra of degree 1). Then it turns out that $\mathrm{Hom}(C^\infty(M,\mathbb{R}),\mathbb{D}) = TM$, the tangent bundle of M. For if φ is a homomorphism $C^\infty(M,\mathbb{R}) \to \mathbb{D}$, then $\pi \circ \varphi : C^\infty(M,\mathbb{R}) \to \mathbb{D} \to \mathbb{R}$ equals ev_x for some $x \in M$ and $\varphi(f) - f(x).1 = X(f).\varepsilon$, where X is a derivation over x since φ is a homomorphism. So X is a tangent vector of M with foot point x. Similarly we may show that $\mathrm{Hom}(C^\infty(M,\mathbb{R}),\mathbb{D}\otimes\mathbb{D}) = TTM$.

Now let A be an arbitrary commutative real finite dimensional algebra with unit. Let $W(A)$ be the subalgebra of A generated by the idempotent and nilpotent elements of A. We will show in this chapter, that $\mathrm{Hom}(C^\infty(M,\mathbb{R}),A) = \mathrm{Hom}(C^\infty(M,\mathbb{R}),W(A))$ is a manifold, functorial in M, and that in this way we have defined a product preserving functor $\mathcal{M}f \to \mathcal{M}f$ for any such algebra. A will be called a *Weil algebra* if $W(A) = A$, since in [Weil, 51] this construction appeared for the first time. We are aware of the fact, that Weil algebras denote completely different objects in the Chern-Weil construction of characteristic classes. This will not cause troubles, and a serious group of mathematicians has already adopted the name Weil algebra for our objects in synthetic differential geometry, so we decided to stick to this name. The functors constructed

in this way will be called Weil functors, and we will also present a covariant approach to them which mimics the construction of the bundles of velocities, due to [Morimoto, 69], cf. [Kolář, 86].

We will discuss thoroughly natural transformations between Weil functors and study sections of them, a sort of generalized vector fields. It turns out that the addition of vector fields generalizes to a group structure on the set of all sections, which has a Lie algebra and an exponential mapping; it is infinite dimensional but nilpotent.

Conversely under very mild conditions we will show, that up to some covering phenomenon each product preserving functor is of this form, and that natural transformations between them correspond to algebra homomorphisms. This has been proved by [Kainz-Michor, 87] and independently by [Eck, 86] and [Luciano, 88].

Weil functors will play an important role in the rest of the book, and we will frequently compare results for other functors with them. They can be much further analyzed than other types of functors.

CHAPTER IX. BUNDLE FUNCTORS ON MANIFOLDS

The description of the product preserving bundle functors on $\mathcal{M}f$ in terms of Weil algebras reflects their general properties in a rather complete way. In the present chapter we use some other procedures to deduce the basic geometric properties of arbitrary bundle functors on $\mathcal{M}f$. Hence the basic subject of this theory is a bundle functor on $\mathcal{M}f$ that does not preserve products. Sometimes we also contrast certain properties of the product-preserving and non-product-preserving bundle functors on $\mathcal{M}f$. First we study the bundle functors with the so-called point property, i.e. the image of a one-point set is a one-point set. In particular, we deduce that their fibers are numerical spaces and that they preserve products if and only if the dimensions of their values behave well. Then we show that an arbitrary bundle functor on manifolds is, in a certain sense, a 'bundle' of functors with the point property. For an arbitrary vector bundle functor F on $\mathcal{M}f$ with the point property we also derive a canonical Lie group structure on the prolongation FG of a Lie group G.

Next we introduce the concept of a flow-natural transformation of a bundle functor F on manifolds. This is a natural transformation $FT \to TF$ with the property that for every vector field $X\colon M \to TM$ its functorial prolongation $FX\colon FM \to FTM$ is transformed into the flow prolongation $\mathcal{F}X\colon FM \to TFM$. We deduce that every bundle functor F on manifolds has a canonical flow-natural transformation, which is a natural equivalence if and only if F preserves products. Then we point out some special features of natural transformations from a Weil functor into an arbitrary bundle functor on $\mathcal{M}f$. This gives a rather effective method for their description. We also deduce that the homotheties are the only natural transformations of the r-th order tangent bundle $T^{(r)}$ into itself. This demonstrates that some properties of $T^{(r)}$ are quite different from those of Weil bundles, where such natural transformations are in bijection with a usually much larger set of all endomorphisms of the corresponding Weil algebras. In the last section we describe basic properties of the so-called star bundle functors, which reflect some constructions of contravariant character on $\mathcal{M}f$.

CHAPTER X.
PROLONGATION OF VECTOR FIELDS AND CONNECTIONS

This section is devoted to systematic investigation of the natural operators transforming vector fields into vector fields or general connections into general connections. For the sake of simplicity we also speak on the prolongations of vector fields and connections. We first determine all natural operators transforming vector fields on a manifold M into vector fields on a Weil bundle over M. In the formulation of the result as well as in the proof we use heavily the technique of Weil algebras. Then we study the prolongations of vector fields to the bundle of second order tangent vectors. We like to comment the interesting general differences between a product-preserving functor and a non-product-preserving one in this case. For the prolongations of projectable vector fields to the r-jet prolongation of a fibered manifold, which play an important role in the variational calculus, we prove that the unique natural operator, up to a multiplicative constant, is the flow operator.

Using the flow-natural equivalence we construct a natural operator transforming general connections on $Y \to M$ into general connections on $T_AY \to T_AM$ for every Weil algebra A. In the case of the tangent functor we determine all first-order natural operators transforming connections on $Y \to M$ into connections on $TY \to TM$. This clarifies that the above mentioned operator is not the unique natural operator in general. Another class of problems is to study the prolongations of connections from $Y \to M$ to $FY \to M$, where F is a functor defined on local isomorphisms of fibered manifolds. If we apply the idea of the flow prolongation of vector fields, we see that such a construction depends on an r-th order linear connection on the base manifold, provided r means the horizontal order of F. In the case of the vertical tangent functor we obtain the operator defined in another way in chapter VII. For the functor J^1 of the first jet prolongation of fibered manifolds we deduce that all natural operators transforming a general connection on $Y \to M$ and a linear connection on M into a general connection on $J^1Y \to M$ form a simple 4-parameter family. In conclusion we study the prolongation of general connections from $Y \to M$ to $VY \to Y$. From the general point of view it is interesting that such an operator exists only in the case of affine bundles (with vector bundles as a special sub case). But we can consider arbitrary connections on them (i.e. arbitrary nonlinear connections in the vector bundle case).

Remarks

Section 42 is based on [Kolář, 88a]. The order estimate in 42.4 follows an idea by [Zajtz, 88b] and the proof of lemma 42.7 was communicated by the second author. The results of section 43 were deduced by [Doupovec, 90]. Section 44 is based on [Kolář, Slovák, 90]. The construction of the connection $\mathcal{F}(\Gamma, \Lambda)$ from 45.4 was first presented in [Kolář, 82b]. Proposition 46.7 was proved by [Doupovec, Kolář, 88]. The relation of proposition 46.10 to Finslerian geometry was pointed out by B. Kis.

CHAPTER XI.
GENERAL THEORY
OF LIE DERIVATIVES

It has been clarified recently that one can define the generalized Lie derivative $\tilde{\mathcal{L}}_{(\xi,\eta)}f$ of any smooth map $f\colon M \to N$ with respect to a pair of vector fields ξ on M and η on N. Given a section s of a vector bundle $E \to M$ and a projectable vector field η on E over a vector field ξ on M, the second component $\mathcal{L}_\eta s\colon M \to E$ of the generalized Lie derivative $\tilde{\mathcal{L}}_{(\xi,\eta)}s$ is called the Lie derivative of s with respect to η. We first show how this approach generalizes the classical cases of Lie differentiation. We also present a simple, but useful comparison of the generalized Lie derivative with the absolute derivative with respect to a general connection. Then we prove that every linear natural operator commutes with Lie differentiation. We deduce a similar condition in the non linear case as well. An operator satisfying the latter condition is said to be infinitesimally natural. We prove that every infinitesimally natural operator is natural on the category of oriented m-dimensional manifolds and orientation preserving local diffeomorphisms.

A significant advantage of our general theory is that it enables us to study the Lie derivatives of the morphisms of fibered manifolds (our feeling is that the morphisms of fibered manifolds are going to play an important role in differential geometry). To give a deeper example we discuss the Euler operator in the higher order variational calculus on an arbitrary fibered manifold. In the last section we extend the classical formula for the Lie derivative with respect to the bracket of two vector fields to the generalized Lie derivatives.

Remarks

The general concept of Lie derivative of a map $f\colon M \to N$ with respect to a pair of vector fields on M and N was introduced by [Trautman, 72]. The operations with linear vector fields from the second half of section 47 were described in [Janyška, Kolář, 82]. In the theory of multilinear natural operators, the commutativity with the Lie differentiation is also used as the starting point, see [Kirillov, 77, 80]. Proposition 48.4 was proved by [Cap, Slovák, 92]. According to [Janyška, Modugno, to appear], there is a link between the infinitesimally natural operators and certain systems in the sense of [Modugno, 87a]. The concept of a sector r-form was introduced in [White, 82].

The Lie derivatives of morphisms of fibered manifolds were studied in [Kolář, 82a] in connection with the higher order variational calculus in fibered manifolds. We remark that a further analysis of formula 49.3.(3) leads to an interesting fact

that a Lagrangian of order at least three with at least two independent variables does not determine a unique Poincaré-Cartan form, but a family of such forms only, see e.g. [Kolář, 84b], [Saunders, 89]. The general bracket formula from section 50 was deduced in [Kolář, 82c].

CHAPTER XII.
GAUGE NATURAL BUNDLES AND OPERATORS

In chapters IV and V we have explained that the natural bundles coincide with the associated fiber bundles to higher order frame bundles on manifolds. However, in both differential geometry and mathematical physics one can meet fiber bundles associated to an 'abstract' principal bundle with an arbitrary structure group G. If we modify the idea of bundle functor to such a situation, we obtain the concept of gauge natural bundle. This is a functor on principal fiber bundles with structure group G and their local isomorphisms with values in fiber bundles, but with fibration over the original base manifold. The most important examples of gauge natural bundles and of natural operators between them are related with principal connections. In this chapter we first develop a description of all gauge natural bundles analogous to that in chapter V. In particular, we prove that the regularity condition is a consequence of functoriality and locality and that any gauge natural bundle is of finite order. We also present sharp estimates of the order depending on the dimensions of the standard fibers. So the r-th order gauge natural bundles coincide with the fiber bundles associated to r-th principal prolongations of principal G-bundles (see 15.3), which are in bijection with the actions of the group $W^r_m G$ on manifolds.

Then we discuss a few concrete problems on finding gauge natural operators. The geometrical results of section 52 are based on a generalization of the Utiyama theorem on gauge natural Lagrangians. First we determine all gauge natural operators of the curvature type. In contradistinction to the essential uniqueness of the curvature operator on general connections, this result depends on the structure group in a simple way. Then we study the differential forms of Chern-Weil type with values in an arbitrary associated vector bundle. We find it interesting that the full list of all gauge natural operators leads to a new geometric result in this case. Next we determine all first order gauge natural operators transforming principal connections to the tangent bundle. In the last section we find all gauge natural operators transforming a linear connection on a vector bundle and a classical linear connection on the base manifold into a classical linear connection on the total space.

Remarks

Our approach to gauge natural bundles and operators generalizes directly the theory of natural bundles. So we also prove the regularity originally assumed in [Eck, 81]. Let us mention that, analogously to chapter XI, we can define

the Lie derivative of sections of gauge natural bundles with respect to the right invariant vector fields on the corresponding principal fiber bundles and then the infinitesimally gauge natural operators. The relation between the gauge naturality and infinitesimal gauge naturality is similar to the case of natural bundles if the gauge group is connected; more information can be found in [Cap, Slovák, 92].

The first application of our methods for finding gauge natural operators was presented in [Kolář, 87b]. The considerations in that paper are restricted to the case the structure group is the general linear group $GL(n)$ in an arbitrary dimension (independent of the dimension of the base manifold), for in such a case one can apply directly the results from chapter VI. [Kolář, 87b] has also determined all $GL(n)$-natural operators transforming a principal connection on a principal bundle P and a classical linear connection on the base manifold into a principal connection on W^1P. The curvature-like operators were found in the special case $G = GL(n)$ in [Kolář, 87b] and the general problem was solved in [Kolář, to appear a]. The greater part of the results from section 52 was deduced in [Kolář, to appear b]. Proposition 53.3 was proved for the special case $G = GL(n)$ in [Kolář, 91], the general result is first presented in this book. Section 54 is based on [Gancarzewicz, Kolář, 91].

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List of symbols

1_j the multi index with j-th component one and all others zero, 13.2
$\alpha\colon J^r(M,N) \to M$ the source mapping of jets, 12.2
$\beta\colon J^r(M,N) \to N$ the target mapping of jets, 12.2
$B\colon \mathcal{FM} \to \mathcal{M}f$ the base functor, 2.20
$C^\infty E$, also $C^\infty(E \to M)$ the space of smooth sections of a fiber bundle
$C^\infty(M,N)$ the space of smooth maps of M into N
$C^\infty_x(M,N)$ the space of germs at $x \in M$, 1.4
$\mathrm{conj}_a\colon G \to G$ the conjugation in a Lie group G by $a \in G$, 4.24
d usually the exterior derivative, 7.8
$\mathbb{D}$ the algebra of dual numbers, 37.1
$\mathbb{D}^r_n = J^r_0(\mathbb{R}^n, \mathbb{R})$ the algebra of r-jets of functions, 40.5
(E,p,M,S), also simply E usually a fiber bundle with total space E, base M, and standard fiber S, 9.1
$\mathcal{F}$ usually the flow operator of a natural bundle F, 6.19, 42.1
Fl^X_t, also $\mathrm{Fl}(t,X)$ the flow of a vector field X, 3.7
$\mathcal{FM}$ the category of fibered manifolds and fiber respecting mappings, 2.20
$\mathcal{FM}_m$ the category of fibered manifolds with m-dimensional bases and fiber respecting mappings with local diffeomorphisms as base maps, 12.16
$\mathcal{FM}_{m,n}$ the category of fibered manifolds with m-dimensional bases and n-dimensional fibers and locally invertible fiber respecting mappings, 17.1
$\mathcal{FM}^*$ the category of star bundles, 41.1
G usually a general Lie group with multiplication $\mu : G \times G \to G$, left translation λ, and right translation ρ
$\mathfrak{g}$ the Lie algebra of a Lie group G
G^r_m the jet group (differential group) of order r in dimension m, 12.6
$G^r_{m,n}$ the jet group of order r of the category $\mathcal{FM}_{m,n}$, 18.8
$GL(n)$ the general linear group in dimension n with real coefficients, 4.30
$GL(\mathbb{R}^n, E)$ the linear frame bundle of a vector bundle E, 10.11
$\mathbb{I}_k$ short for the $k \times k$-identity matrix $\mathrm{Id}_{\mathbb{R}^k}$
$\mathrm{inv}J^r(M,N)$ the bundle of invertible r-jets of M into N, 12.3
$J^r E$ the bundle of r-jets of local sections of a fiber bundle $E \to M$, 12.16
$J^r(M,N)$ the bundle of r-jets of smooth functions from M to N, 12.2
$j^r f(x)$, also $j^r_x f$ the r-jet of a mapping f at x, 12.2
K^r_n the functor of (n,r)-contact elements, 12.15
$\mathcal{L}$ the Lie derivative, 6.15, 47.4
$\ell\colon G \times S \to S$ usually a left action of a Lie group
$L(V,W)$ the space of all linear maps of vector space V into a vector space W
$LP = P[\mathfrak{g}, \mathrm{Ad}]$ the adjoint bundle of principal bundle $P(M,G)$, 17.6
L^r the r-th order skeleton of the category $\mathcal{M}f$, 12.6
M usually a (base) manifold
$\mathcal{M}f$ the category of manifolds and smooth mappings, 1.2
$\mathcal{M}f_m$ the category of m-dimensional manifolds and local diffeomorphisms, 6.14

$\mathbb{N}$ natural numbers
$\Omega^k(M)$ the space of k-forms on a manifold M, 7.4
$\Omega^k(M,E)$ the space of E-valued k-forms, 7.11
$P(M,G)$, also (P,p,M,G) a principal fiber bundle with structure group G, 10.6
$P[S,\ell]$, also $P[S]$ the associated bundle to a principal bundle $P(M,G)$ for the action $\ell\colon G\times S\to S$, 10.7
$\mathcal{PB}$ the category of principal fiber bundles, 10.6
$\mathcal{PB}_m$ the category of principal bundles over m-dimensional manifolds and of $\mathcal{PB}$-morphisms covering local diffeomorphisms, 17.4
$\mathcal{PB}(G)$ the category of principal G-bundles, 10.6
$\mathcal{PB}_m(G)$ the category of principal G-bundles over m-dimensional manifolds and local isomorphisms, 15.1
$P^rM = \mathrm{inv}J^r_0(\mathbb{R}^{\dim M},M)$ the r-th order frame bundle of a manifold M, 12.12
$\pi^r_s : J^r(M,N)\to J^s(M,N)$ projection of r-jets into s-jets, $s\le r$, 12.2
QP the connection bundle of a principal bundle P, 17.4
$Q_\tau P^1M$ the bundle of torsion free linear connections, 25.3
$\mathbb{R}$ real numbers
$r\colon P\times G\to P$ usually a right action, in particular the principal right action of a principal bundle
TM the tangent bundle of a manifold M with projection $\pi_M : TM\to M$ 1.7
$T^{(r)}M = J^r(M,\mathbb{R})^*_0$ the r-order tangent bundle, 12.14
$T^r_k = J^r_0(\mathbb{R}^k,\ )$ the functor of (k,r)-velocities, 12.8
T_A the Weil functor corresponding to the Weil algebra A, 35.11
t_x usually the translation $\mathbb{R}^m\to\mathbb{R}^m$, $y\mapsto y+x$
$\mathcal{VB}$ the category of vector bundles, 6.3
W^r_mG the (m,r)-principal prolongation of a Lie group G, 15.2
W^rP the r-th principal prolongation of a principal bundle P, 15.3
$\mathfrak{X}(M)$ the set of all vector fields on a manifold M, 3.1
$Y\to M$ usually a fibered manifold
$\mathbb{Z}$ integers

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